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III. Approximate Construction for an Ellipse.

By J. W. Bradshaw, University of Michigan.

In view of the approximate construction for an ellipse given by Mr. Richard Iwerson in the November number of the Monthly, 1916, pp. 354–355, and discussed by Mr. Paul Capron in the number for February, 1917, pp. 90–93, readers of the Monthly may be interested in a few observations on other solutions of this problem. Several such have been published, to six of which I shall refer as French's[1] first, second, and third, Richter's,[2] Honey's,[3] and Clark's[4] constructions. French's first and third and Richter's replace the quadrant of the ellipse by two circular arcs; French's second, Honey's, and Clark's by three. In all of these the approximate quadrant passes through the ends of the axes and the circular arcs are tangent to each other where they meet.

Two-arc constructions of the quadrant. We may think of the two-arc approximation as arising from an alteration of the major and minor circles of curvature until they become tangent to each other. In Fig. 1 let $OA = a$ and $OB = b$ be the semi-axes of the ellipse, which for the sake of brevity of statement we refer to coördinate axes through its center. Let C_1 and C_2 be the centers of curvature for the points A and B; they lie on the perpendicular to the line AB through the point $K = (a, b)$. Let ρ_1 and ρ_2 be the corresponding radii of curvature, σ_1 and σ_2 their altered values, and D_1 and D_2 the corresponding centers. The condition that the two arcs where they meet shall be tangent requires that the centers D_1 and D_2 and the point of meeting M shall be collinear. The right triangle OD_1D_2 gives

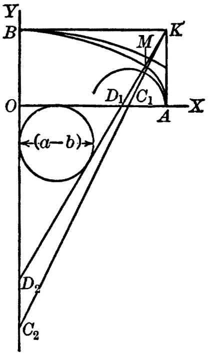

Fig. 1.

$$(\sigma_2 - b)^2 + (a - \sigma_1)^2 = (\sigma_2 - \sigma_1)^2,$$

From this it is easy to show that the line D_1D_2 is a tangent of the circle whose equation is

$$\left(x - \frac{a-b}{2}\right)^2 + \left(y + \frac{a-b}{2}\right)^2 = \left(\frac{a-b}{2}\right)^2.$$

French's first and third and Richter's constructions use different tangents of this circle. The last named possesses the advantage that the point M lies on the true ellipse. The condition for this is that the line D_1D_2 shall pass through the point K. We have then the following very simple statement for the construction of this approximation:

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[1] Thomas E. French, *Engineering Drawing*, pp. 53, 54.

[2] Otto Richter, Der Ellipsenreif, *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, Vol. 42 (1911), p. 583.

[3] Frederic R. Honey, "A Method of Constructing an Ellipse and Measuring the Curved Length," *Engineering News*, April, 1907, p. 388.

[4] J. J. Clark, "A Method of Constructing an Approximate Ellipse with Three Radii," *Engineering News*, Nov., 1911, p. 627.

From the point (a, b) draw that one of the two tangents of the fourth-quadrant circle of diameter a–b touching the axes which is farther from the origin. This tangent cuts the axes in the centers D_1 and D_2 sought.

While Richter's construction yields the same approximation, its execution is less simple.

Three-arc Constructions of the Quadrant.—The three-arc constructions may be thought of as derived from three circles of curvature, the major, minor, and some intermediate one. There is considerable latitude in altering these so that the intermediate circle shall be tangent to the others. French's second construction uses the major and minor circles of curvature without alteration, the radius of the intermediate circle being the mean proportional between a and b, Clark's uses the major circle, Honey's alters both the major and the minor circles and uses the arithmetic mean between a and b as the radius of the intermediate circle. For most purposes a construction would seem best which alters all three circles. The following is suggested as typical:

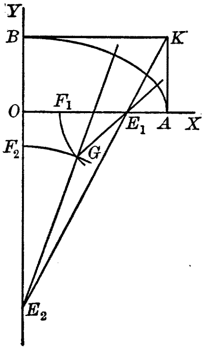

FIG. 2.

In Fig. 2 let E_1E_2 be some line between the lines C_1C_2 and D_1D_2 of Fig. 1—*e. g.*, the bisector of the angle formed by these lines. With a convenient radius—*e. g.*, the mean proportional between a and b—lay off AF_1 and BF_2. With E_1 and E_2 as centers and radii E_1F_1 and E_2F_2 strike arcs intersecting in G. E_1, G, and E_2 are the desired centers and the radii are E_1A, $F_1A = F_2B$, and E_2B.

It would be difficult to give a categorical answer to the question, "What criterion best measures the degree of approximation attained?" The purpose for which the curve is to be used would influence the choice of such a criterion. Four criteria suggest themselves to me: the total area between the curve and the approximation, the difference in length of the two curves, the maximum distance between corresponding points, where by corresponding points I mean those lying on a normal to the ellipse, the maximum angle between the tangents at corresponding points. Doubtless many others might be suggested.

A CORRECTION BY P. J. DANIELL.

TO THE EDITORS: Will you oblige me by publishing this notice of an error in my paper on "New Rules of Quadrature" in the March issue?

Rule 3 is not new but was given by S. A. Corey, in the MONTHLY for June, 1912, formula 25*s*. His error term is too large but mine was incorrect. .In my paper should be read on page 112:

$$\psi_1(x) = z^2(z + \tfrac{1}{2}h^2), \quad \psi_2(x) = z_2^2(z_2 + 2h^2),$$

$$\psi_2(x) - 16\psi_1(x) = t^4(15t^2 - 42t + 30),$$

$$| S' | \leq \frac{8}{15}\frac{h^6}{7!}(b - a) \max | y^{\mathrm{VI}}(x) |$$